Linear Algebra 2: Tutorial 10

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Exercise 1. *Are all triangular matrices are diagonalizable? Prove your answer.*

Exercise 2. *For any subspace V of $\mathbb{R}^n$, let P_V be the matrix of orthogonal projection onto V. Is P_V orthogonally diagonalizable for every subspace V of $\mathbb{R}^n$? If so, explain why. If not, decide if it must be diagonalizable.*

Exercise 3. *Diagonalize the following matrix in $\mathbb{R}^{n\times n}$ ($n \geq 2$).*

$$A_n = \begin{bmatrix} 3 & 1 & \dots & 1 \\ 1 & 3 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 3 \end{bmatrix}.$$

> ***Hint:*** *Guess the eigenvalues, and then find bases for the corresponding eigenspaces. As a starting point, here are the characteristic polynomials of A_n for small values of n:*
>
> - $p_{A_2}(\lambda) = (\lambda - 4)(\lambda - 2)$
> - $p_{A_3}(\lambda) = (\lambda - 5)(\lambda - 2)^2$
> - $p_{A_4}(\lambda) = (\lambda - 6)(\lambda - 2)^3$

Exercise 4. *Let p be any real number. Diagonalize the following matrix in $\mathbb{R}^{n\times n}$ ($n \geq 2$).*

$$B_n = \begin{bmatrix} p & 1 & \dots & 1 \\ 1 & p & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & p \end{bmatrix}.$$

Exercise 5. *Orthogonally diagonalize the following matrix in* $\mathbb{R}^{n\times n}$ *(*$n \geq 2$*).*

$$C_n \quad = \quad \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix}.$$

> ***Hint:*** *To make your* Q*, you might want to start with vectors* $\mathbf{w}_k := \begin{bmatrix} 1 & \dots & 1 & -k & 0 & \dots & 0 \end{bmatrix}^T$ *(*k *many* 1*'s), plus another vector (which one?). Now what?*

Exercise 6. *Suppose that* $A, B \in \mathbb{R}^{n\times n}$ *are symmetric. Show that* AB *is symmetric if and only if* $AB = BA$*.*

Exercise 7. *Suppose that* $A, B \in \mathbb{R}^{n\times n}$ *are orthogonally diagonalizable. When is* AB *orthogonally diagonalizable?*