# Linear Algebra 2: Tutorial 9

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**Exercise 1.** *Let $\lambda_0$ be a scalar in a field $\mathbb{F}$, and let $k$ be a positive integer. Consider the following matrix in $\mathbb{F}^{k\times k}$:*

$$J_k(\lambda_0) \quad := \quad \begin{bmatrix} \lambda_0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & \lambda_0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \lambda_0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \lambda_0 & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & \lambda_0 \end{bmatrix}.$$

*Compute the characteristic polynomial of $J_k(\lambda_0)$, all the eigenvalues of $J_k(\lambda_0)$ along with their algebraic and geometric multiplicities, as well as a basis for each eigenspace.*

**Exercise 2.** *Consider the following matrix in $\mathbb{R}^{3\times 3}$:*

$$A \quad = \quad \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$$

*This matrix can be written in the form*

$$A \quad = \quad \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/4 & 1/2 & -3/4 \\ 1/4 & -1/2 & 1/4 \end{bmatrix},$$

*where the first and third matrix on the right-hand-side are each other's inverses. Compute the characteristic polynomial $p_A(\lambda)$ of $A$, all the eigenvalues of $A$ along with their algebraic and geometric multiplicities, as well as a basis for each eigenspace of $A$. Can you compute this just by looking at the decomposition above (i.e. without actually computing)?*

**Exercise 3.** *Consider the matrix $A$ from Exercise 2. Find a formula for $A^m$ for a non-negative integer $m$. What happens if $m$ is a negative integer?*

**Exercise 4.** *For each of the following matrices $A$ (with entries understood to be in $\mathbb{C}$), determine whether the matrix is diagonalizable, and if so, diagonalize it and find a formula for $A^n$.*

*(a)* $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$;

*(b)* $A = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix}$;

*(c)* $A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix}$;

*(d)* $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$.