Linear Algebra 2: Tutorial 5

Irena Penev

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Exercise 5 from Tutorial 4. *Using the least-squares method, fit a quadratic function to the four data points* $(a_1, b_1) = (-1, 8)$, $(a_2, b_2) = (0, 8)$, $(a_3, b_3) = (1, 4)$, *and* $(a_4, b_4) = (2, 16)$.

Exercise 6 from Tutorial 4. *Explain how one would fit a trigonometric function of the form* $f(t) = c_0 + c_1 \sin t + c_2 \cos t$ *that best fits the data points* $(0, 0)$, $(1, 1)$, $(2, 2)$, $(3, 3)$.

> ***Remark:*** *So, you should set up the matrix-vector equation* $A\mathbf{c} = \mathbf{b}$ *(where* A *and* $\mathbf{b}$ *are known and* $\mathbf{c}$ *is the unknown) to which the least-squares method should be applied. Don't compute the actual least-squares solution.*

Exercise 1. *Let* $a, b, c, d, e, f, g, h, i$ *be real numbers such that*

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7.$$

Compute the following determinants.

1. $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix}$

2. $\begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix}$

3. $\begin{vmatrix} a & b & c \\ g & h & i \\ d & e & f \end{vmatrix}$

4. $\begin{vmatrix} g & h & i \\ a & b & c \\ d & e & f \end{vmatrix}$

5. $\begin{vmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{vmatrix}$

6. $\begin{vmatrix} a+d & b+e & c+f \\ d & e & f \\ g & h & i \end{vmatrix}$

Exercise 2. *Compute the determinants of the matrices below, with entries understood to be in* $\mathbb{R}$.

- $A_1 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$
- $A_2 = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 4 & 6 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$
- $A_3 = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & 0 & -2 & 4 \\ 3 & 3 & -3 & 3 \\ 4 & 4 & -4 & 4 \end{bmatrix}$
- $A_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$
- $A_5 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{bmatrix}$
- $A_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 & 5 \end{bmatrix}$

Exercise 3. *Let A be an $n \times n$ matrix of the following form:*

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 & a_{1,n} \\ 0 & 0 & \dots & a_{2,n-1} & a_{2,n} \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & a_{n-1,2} & \dots & a_{n-1,n-1} & a_{n-1,n} \\ a_{n,1} & a_{n,2} & \dots & a_{n,n-1} & a_{n,n} \end{bmatrix}$$

Find a formula for det(A).

Exercise 4. *Compute the determinant below.*

$$\begin{vmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 4 & 0 \end{vmatrix}$$

Exercise 5. *Determine whether the determinant below is positive, negative, or zero. Can you tell which it is without actually computing the determinant?*

$$\begin{vmatrix} 1 & 1000 & 2 & 3 \\ 1000 & -3 & 5 & 0 \\ 2 & 3 & 5 & 1000 \\ 1 & 2 & 1000 & 4 \end{vmatrix}$$

Theorem 5.1 from Lecture Notes 15. *Let* $A \in \mathbb{F}^{n\times n}$*. Then* A *is invertible if and only if* $det(A) \neq 0$*.*

Exercise 6. *Using determinants, determine for which (if any) values of* k *the matrix* A *(below) is invertible. (Entries of* A *are understood to be in* $\mathbb{R}$*).*

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & k & 1 \\ 3 & 1 & 0 \end{bmatrix}$$