Linear Algebra 2:
HW#7

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due Friday, May 12, 2022, at 10 am (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Alternatively (if you don't feel like typing or scanning), you may submit a hard copy of your HW in lecture or tutorial **before** the deadline. Please do **not** e-mail your HW to me. Please write your **name** on the top of the first page of your HW.

Problem 1 (30 points)**.** *Let* $\langle \cdot, \cdot \rangle : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ *be given by*

$$\langle \mathbf{x}, \mathbf{y} \rangle \quad = \quad 2x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2 - x_2y_3 - x_3y_2 + 2x_3y_3$$

for all $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$ *and* $\mathbf{y} = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$ *in* $\mathbb{R}^3$*. Determine whether* $\langle \cdot, \cdot \rangle$ *is a scalar product in* $\mathbb{R}^3$*. Make sure you justify your answer.*

Problem 2 (30 points)**.** *Let* $A \in \mathbb{R}^{n \times n}$ *be a positive definite matrix. Prove that there exists a positive definite matrix* $B \in \mathbb{R}^{n \times n}$ *such that* $A = B^2$*.*

Problem 3 (40 points)**.** *Let* A *and* B *be symmetric matrices in* $\mathbb{R}^{n \times n}$*, and assume that every eigenvalue of* A *is strictly greater than every eigenvalue of* B*. Prove that the matrix* $A - B$ *is positive definite.*

> ***Hint:*** *Both the spectral theorem and the Pythagorean theorem are likely to be useful.*