Linear Algebra 2:
HW#6

Irena Penev
Winter 2023

due Friday, April 28, 2022, at 10 am (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Alternatively (if you don't feel like typing or scanning), you may submit a hard copy of your HW in lecture or tutorial **before** the deadline. Please do **not** e-mail your HW to me. Please write your **name** on the top of the first page of your HW.

Problem 1 (20 points)**.** *Prove or disprove the following statement:*

> *If* $\mathbf{e}_1, \dots, \mathbf{e}_n$ *are eigenvectors of a matrix* $A \in \mathbb{C}^{n \times n}$*, then* A *must be diagonal.*

Remark: *First, state clearly whether the statement is true or false. If it is true, prove it. If it is false, construct a counterexample (and prove that your counterexample really is a counterexample).*

Problem 2 (20 points)**.** *Let* $\mathbb{F}$ *be a field, and let* $A \in \mathbb{F}^{n \times n}$ *be a matrix. Prove that* A *and* A^T *have the same characteristic polynomial.*

Problem 3 (20 points)**.** *Let* $\mathbb{F}$ *be a field, and let* $A \in \mathbb{F}^{n \times n}$ *be a matrix that has* n *linearly independent eigenvectors. Prove that* A^T *also has* n *linearly independent eigenvectors.*

> ***Hint:*** *Diagonalization.*

Problem 4 (20 points)**.** *Consider the matrix*

$$A = \begin{bmatrix} 7 & 4 & 16 \\ 2 & 5 & 8 \\ -2 & -2 & -5 \end{bmatrix}$$

in $\mathbb{C}^{3\times 3}$*. Diagonalize the matrix* A*, and find a formula for* A^m *(for a non-negative integer* m*). Does your formula also work for negative integers* m*? Explain your answer.*

Problem 5 (20 points)**.** *Orthogonally diagonalize the following symmetric matrix in* $\mathbb{R}^{4\times 4}$*.*

$$A = \begin{bmatrix} 4 & 1 & 3 & 1 \\ 1 & 4 & 1 & 3 \\ 3 & 1 & 4 & 1 \\ 1 & 3 & 1 & 4 \end{bmatrix}$$