

Lineární algebra 1: Opačné a inverzní prvky v $\mathbb{Z}_p$ ($p = 2, 3, 5$)

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Opačné prvky:

- v $\mathbb{Z}_2$: $-0 = 0$, $-1 = 1$;
- v $\mathbb{Z}_3$: $-0 = 0$, $-1 = 2$, $-2 = 1$;
- v $\mathbb{Z}_5$: $-0 = 0$, $-1 = 4$, $-2 = 3$, $-3 = 2$, $-4 = 1$.

Inverzní prvky:

- v $\mathbb{Z}_2$: $1^{-1} = 1$;
- v $\mathbb{Z}_3$: $1^{-1} = 1$, $2^{-1} = 2$;
- v $\mathbb{Z}_5$: $1^{-1} = 1$, $2^{-1} = 3$, $3^{-1} = 2$, $4^{-1} = 4$.