Linear Algebra 1: Tutorial 2

Irena Penev

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$$\begin{bmatrix} 0 & \blacksquare & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & \blacksquare & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \blacksquare & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & * & 0 & 0 & * & * & 0 & * & * \\ 0 & 0 & 0 & 1 & 0 & * & * & 0 & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

REF RREF

Addition and multiplication tables for $\mathbb{Z}_p$ ($p = 2, 3, 5$):

$\mathbb{Z}_2$:

+	0	1
0	0	1
1	1	0

·	0	1
0	0	0
1	0	1

$\mathbb{Z}_3$:

+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

·	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

$\mathbb{Z}_5$:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

·	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Exercise 1. *Compute the reduced row echelon form of the following matrices. What are the pivot columns and the pivot positions of these matrices?*

(a) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$, *with entries understood to be in* $\mathbb{R}$*;*

(b) $\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{R}$*;*

(c) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_2$*;*

(d) $\begin{bmatrix} 2 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_3$*;*

(e) $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 4 & 3 \\ 4 & 4 & 0 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_5$*.*

Exercise 2. *Solve the following linear system, with coefficients understood to be in* $\mathbb{Z}_2$*. How many solutions does the linear system have?*

$$\begin{array}{ccccccccc} x_1 & + & x_2 & + & x_3 & + & x_4 & = & 0 \\ & & x_2 & + & x_3 & & & = & 0 \\ x_1 & + & x_2 & & & + & x_4 & = & 1 \\ x_1 & & & & & + & x_4 & = & 0 \end{array}$$

Exercise 3. *Solve the following linear system, with coefficients understood to be in* $\mathbb{Z}_3$*. How many solutions does the linear system have?*

$$\begin{array}{ccccccccc} x_1 & + & 2x_2 & & & + & x_4 & = & 1 \\ & & x_2 & + & 2x_3 & & & = & 0 \\ x_1 & + & x_2 & + & x_3 & + & x_4 & = & 2 \end{array}$$

Exercise 4. *Solve the following linear system, with coefficients understood to be in* $\mathbb{R}$*. How many solutions does the linear system have?*

$$\begin{array}{ccccccccccccc} 3x_1 & - & 6x_2 & + & 2x_3 & - & 3x_4 & + & 4x_5 & & & = & 8 \\ 2x_1 & - & 4x_2 & + & x_3 & - & 3x_4 & + & 2x_5 & & & = & 5 \\ 3x_1 & - & 6x_2 & + & x_3 & - & 6x_4 & + & 2x_5 & + & x_6 & = & 4 \\ x_1 & - & 2x_2 & & & - & 3x_4 & & & & & = & 2 \end{array}$$