# Linear Algebra 1: HW#7

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due Friday, December 6, 2024, at noon (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and it should be either typed or written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Please do **not** send your HW by e-mail. Please write your **name** on top of the first page of your HW.

Unless explicitly stated otherwise, make sure you **show your work** (including any row reduction that you may need to perform in order to solve the exercise/problem). If you do not show your work, you will receive **no credit** for the exercise/problem in question, even if your final answer is correct.

**Remark:** Feel free to use a calculator such as the one from

`https://www.dcode.fr/matrix-row-echelon`

for any row reduction that you need to perform. However, keep in mind that on the exam, you might be asked to solve problems like this **without** a calculator, and so you should in principle be able to perform row reduction of this sort by hand.[a] Moreover, make sure you correctly tell the calculator what kinds of numbers you are working with,[b] because otherwise, the calculator may give you a wrong answer.

[a] If you do not yet feel confident in your ability to row reduce, then row reduce by hand first, and then check your answer with a calculator.

[b] For real numbers, you should use their "Echelon Form Matrix Reduction Calculator." For $\mathbb{Z}_2$, use "RREF with Modulo Calculator" with Base/Modulus = 2. For $\mathbb{Z}_3$, use "RREF with Modulo Calculator" with Base/Modulus = 3.

**Problem 1** (30 points)**.** *Prove parts (b), (c), and (d) of Proposition 3.1.3 of the Lecture Notes (see page 214).*

**Problem 2** (70 points)**.** *Let $V$ be a vector space over a field $\mathbb{F}$, and let $U_1$ and $U_2$ be subspaces of $V$.*

*(a) [20 points] Prove that $U_1 \cap U_2$ is a subspace of $V$.*

*(b) [20 points] Prove that $U_1 + U_2 := \{\mathbf{u}_1 + \mathbf{u}_2 \mid \mathbf{u}_1 \in U_1, \mathbf{u}_2 \in U_2\}$ is a subspace of $V$.*

*(c) [30 points] Prove that $U_1 \cup U_2$ is a subspace of $V$ if and only if one of $U_1, U_2$ is included in the other.*[1]

***Hint (for all three parts):*** *Theorem 3.1.7 of the Lecture Notes.*

[1] "One of $U_1, U_2$ is included in the other" means "$U_1 \subseteq U_2$ or $U_2 \subseteq U_1$."