Linear Algebra 1: HW#2

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due Friday, October 18, 2024, at noon (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and it should be either typed or written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Please do **not** send your HW by e-mail. Please write your **name** on top of the first page of your HW.

Unless explicitly stated otherwise, make sure you **show your work** (including any row reduction that you may need to perform in order to solve the exercise/problem). If you do not show your work, you will receive **no credit** for the exercise/problem in question, even if your final answer is correct.

Exercise 1 (30 points)**.** *Compute the reduced row echelon (RREF) form of each of the following matrices.*

(a) $A = \begin{bmatrix} 2 & -4 & -1 & 5 \\ 3 & -6 & 1 & 0 \\ -1 & 2 & 0 & -1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{R}$*;*

(b) $B = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_2$*;*

(c) $C = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_3$*.*

Exercise 2 (20 points)**.** *Solve the following system of linear equations, with coefficients understood to be in* $\mathbb{Z}_3$*. How many solutions does the system have?*

$$\begin{array}{rcrcrcrcrcl} 2x_1 & + & x_2 & + & 2x_3 & + & 2x_4 & + & x_5 & = & 0 \\ x_1 & + & 2x_2 & + & 2x_3 & + & 2x_4 & + & x_5 & = & 2 \\ 2x_1 & + & x_2 & & & & & + & 2x_5 & = & 2 \end{array}$$

Problem 1 (30 points). *Solve the linear system below (with coefficients understood to be in $\mathbb{R}$), and specify how many solutions it has.*

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & 5x_3 & = & -3 \\ x_1 & & & + & (k^2-4k-2)x_3 & = & k-4 \\ x_1 & + & 2x_2 & + & (k^2-4k)x_3 & = & k-8 \end{array}$$

Here, k is a parameter (which should be treated as a fixed real constant), and x_1, x_2, x_3 are the variables.

> ***Remark:*** *Your solutions will depend on k. In fact, the* ***number*** *of solutions will also depend on k. You will need to figure out for which (if any) k the system has no solutions, for which (if any) it has a unique solution, and for which (if any) it has infinitely many solutions. If the system is consistent, make sure you specify the solutions!*
>
> ***Hint:*** *This is similar to Example 1.3.18 of the Lecture Notes.*

Problem 2 (20 points). *For which (if any) values of the real parameter k are the real matrices*

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} k & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \\ 2 & 3 & 4 & 5 \end{bmatrix}$$

row equivalent? Make sure you prove that your answer is correct.

> ***Hint:*** *There is a result in the Lecture Notes that tells you when two matrices are row equivalent. Which result (proposition/lemma/theorem/corollary) is that?*