Linear Algebra 1: Tutorial 7

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Exercise 1. *Consider the following permutation in S_5:*

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 1 & 2 \end{pmatrix}.$$

(a) Find the disjoint cycle decomposition of π.

(b) Express π as a composition of transpositions.

(c) Find all the inversions of π.

(d) Compute the sign of π in three different ways:

- *using the definition (i.e. using the disjoint cycle decomposition of π),*
- *using transpositions,*
- *using inversions.*

Exercise 2. *Consider the following permutation in S_7:*

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 2 & 3 & 7 & 5 & 6 \end{pmatrix}.$$

(a) Find the disjoint cycle decomposition of π.

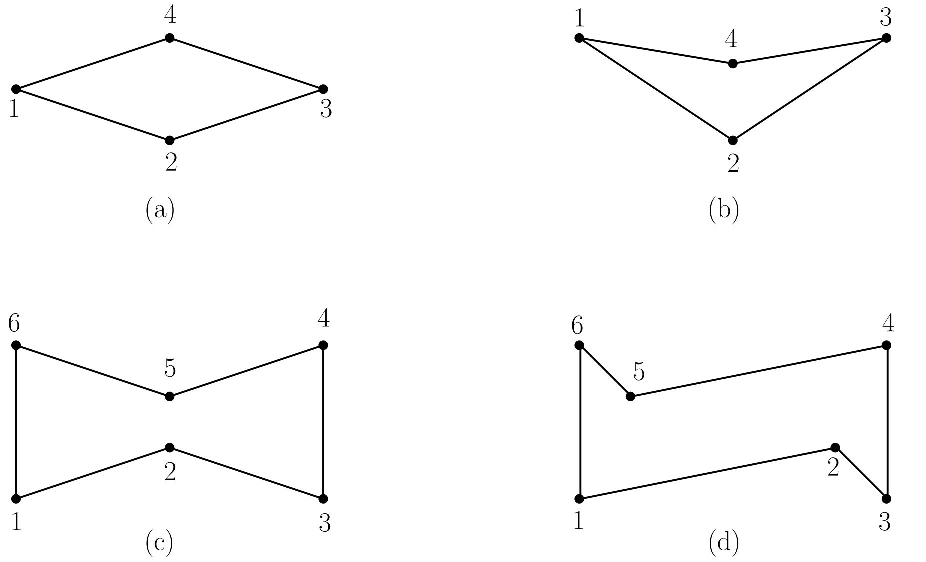
(b) Express π as a composition of transpositions.

(c) Find all the inversions of π.

(d) Compute the sign of π in three different ways:

- *using the definition (i.e. using the disjoint cycle decomposition of π),*
- *using transpositions,*
- *using inversions.*

Exercise 3. *Compute the group of symmetries for each of the polygons below.*



Exercise 4. *Let n and k be positive integers such that $k \leq n$. What is the probability that in a random permutation in S_n, the number 1 is in a cycle of length k?*