Linear Algebra 1: Tutorial 6

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Exercise 3 from Tutorial 5. *Construct two non-zero matrices (i.e. matrices that have at least one non-zero entry), A and B, such that AB is a zero matrix (i.e. matrix with all zero entries). Can you choose A and B so that all entries of A and B are non-zero?*

Exercise 4 from Tutorial 5. *Construct a matrix $A \in \mathbb{R}^{2\times 4}$ and a matrix $B \in \mathbb{R}^{4\times 2}$ such that $AB = I_2$. Now for your matrices A and B, compute the product BA. Note that you do **not** get I_4.*

Exercise 5 from Tutorial 5. *Construct two invertible matrices $A, B \in \mathbb{R}^{2\times 2}$ such that $A + B$ is **not** invertible.*

Exercise 1. *Find the disjoint cycle decompositions of the permutations below. In each case, find both the disjoint cycle decomposition with all cycles of length one included, as well as the disjoint cycle decomposition with all cycles of length one omitted.*

(a) $\pi_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 1 & 4 & 7 & 5 & 6 \end{pmatrix}$

(b) $\pi_2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 2 & 1 & 3 & 4 & 6 & 8 & 12 & 11 & 10 & 9 & 7 & 5 \end{pmatrix}$.

Exercise 2. *Find the table representation of the following permutations.*

(a) $\sigma_1 = (136)(4)(57)$ *in* S_7*;*

(b) $\sigma_2 = (1342675)$ *in* S_7*;*

(c) $\sigma_3 = (1, 12, 4, 3)(2, 5)(6, 8)(10, 11)$ *in* S_{13}.

Exercise 3. *Compute $\sigma_1 \circ \sigma_2$ and $\sigma_2 \circ \sigma_1$, where σ_1 and σ_2 are as in Exercise 2.*

Exercise 4. *Find the inverses of all the permutations from Exercises 1 and 2. Write each inverse in the same form (table or disjoint cycle) as the original permutation.*

Exercise 5. *Find the sign of the permutations from Exercises 1 and 2.*