# Linear Algebra 1: Tutorial 5

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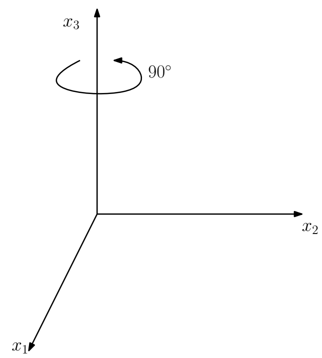
**Exercise 1.** *Find the standard matrices of the following linear functions (you may assume they are linear).*

*(a)* $f : \mathbb{R}^3 \to \mathbb{R}^3$ *that rotates each vector about the* $x_3$*-axis by 90° counterclockwise.*

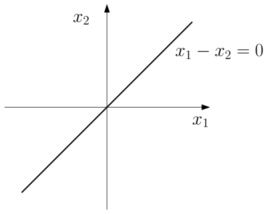



*(b)* $g : \mathbb{R}^3 \to \mathbb{R}^3$ *that projects each vector onto the* $x_1x_3$*-plane.*

*(c)* $h_1 : \mathbb{R}^3 \to \mathbb{R}^3$ *that first rotates each vector about the* $x_3$*-axis by 90° counterclockwise, and then projects each vector onto the* $x_1x_3$*-plane.*

*(d)* $h_2 : \mathbb{R}^3 \to \mathbb{R}^3$ *that first projects each vector onto the* $x_1x_3$*-plane, and then rotates each vector about the* $x_3$*-axis by 90° counterclockwise.*

**Exercise 2.** *Find the standard matrix of the linear function* $f : \mathbb{R}^2 \to \mathbb{R}^2$ *that projects each vector onto the line given by the equation* $x_1 - x_2 = 0$ *(you may assume the function is linear).*



**Exercise 3.** *Construct two non-zero matrices (i.e. matrices that have at least one non-zero entry), $A$ and $B$, such that $AB$ is a zero matrix (i.e. matrix with all zero entries). Can you choose $A$ and $B$ so that all entries of $A$ and $B$ are non-zero?*

**Exercise 4.** *Construct a matrix $A \in \mathbb{R}^{2\times 4}$ and a matrix $B \in \mathbb{R}^{4\times 2}$ such that $AB = I_2$. Now for your matrices $A$ and $B$, compute the product $BA$. Note that you do **not** get $I_4$.*

**Exercise 5.** *Construct two invertible matrices $A, B \in \mathbb{R}^{2\times 2}$ such that $A+B$ is **not** invertible.*

**Exercise 6.** *Consider the following **elementary** matrices (with entries understood to be in $\mathbb{R}$.*

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

*For each of the matrices above, determine which elementary row operation it corresponds to, and find the inverse of the matrix. (You should be able to find the inverse at a glance, without any row reducing.)*

**Exercise 7.** *Consider the matrix below, with entries understood to be in $\mathbb{R}$.*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

*Either express $A$ as a product of elementary matrices, or prove that this is not possible.*