

# Linear Algebra 1: Tutorial 2

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**Exercise 1.** Find the reduced row echelon form of the following matrices. What are the pivot columns and the pivot positions of these matrices?

$$(a) \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}, \text{ with entries understood to be in } \mathbb{R};$$

$$(b) \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}, \text{ with entries understood to be in } \mathbb{R};$$

$$(c) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \text{ with entries understood to be in } \mathbb{Z}_2;$$

$$(d) \begin{bmatrix} 2 & 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 \\ 2 & 2 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix}, \text{ with entries understood to be in } \mathbb{Z}_3;$$

$$(e) \begin{bmatrix} 2 & 4 & 2 \\ 1 & 4 & 3 \\ 4 & 4 & 0 \end{bmatrix}, \text{ with entries understood to be in } \mathbb{Z}_5.$$