Linear Algebra 1: HW#7

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due Friday, ~~December 22~~ December 29, 2023, at noon (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and written in dark ink/pencil (blue, black...) on a light (white, beige...) background. (Of course, you may also type, especially if you know how to use LaTeX.) Other formats will **not** be accepted. Please write your **name** on top of the first page of your HW.

Problem 1 (25 points)**.** *Prove Proposition 3.2.12 from the Lecture Notes (on page 227).*

Problem 2 (50 points)**.** *Read subsection 3.2.6 (on pages 236-238), and then complete the proofs of Proposition 3.2.22 and Theorem 3.2.23, as follows.*

(a) Complete the proof of Proposition 3.2.22 by showing that the set

$$\Big\{(\mathbf{u}_1, \mathbf{0}_W), \dots, (\mathbf{u}_m, \mathbf{0}_W), (\mathbf{0}_U, \mathbf{w}_1), \dots, (\mathbf{0}_U, \mathbf{w}_n)\Big\}$$

is indeed a basis of the vector space $U \times W$.

(b) Complete the proof of Theorem 3.2.23 by showing that the set

$$\Big\{\mathbf{v}_1, \dots, \mathbf{v}_p, \mathbf{u}_1, \dots, \mathbf{u}_{m-p}, \mathbf{w}_1, \dots, \mathbf{w}_{n-p}\Big\}$$

is indeed a basis of $U + W$.

Problem 3 (25 points)**.** *Prove Theorem 3.2.24 from the Lecture Notes (on page 238).*