

Linear Algebra 1: Tutorial 8

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Exercise 1. Let \mathbb{F} be a field.

- (a) Find a basis and the dimension of $\mathbb{F}^{2 \times 3}$.
- (b) Generalize your answer from (a) to $\mathbb{F}^{n \times m}$.
- (c) Find a basis and the dimension of $\mathbb{P}_{\mathbb{F}}^3$ (i.e. the space of polynomials of degree at most 3, with coefficients in \mathbb{F}).
- (d) Generalize your answer from (c) to $\mathbb{P}_{\mathbb{F}}^n$.
- (e) Suppose that \mathbb{F} is finite, and V is an n -dimensional vector space over \mathbb{F} . What is the size of V , i.e. how many vectors are there in V ?

Exercise 2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

with entries understood to be in \mathbb{Z}_2 .

- (a) Compute $\text{rank}(A)$.
- (b) Find a basis for $\text{Col}(A)$.
- (c) Find a basis for $\text{Row}(A)$.
- (d) Find a basis for $\text{Nul}(A)$.

Exercise 3. Let \mathbb{F} be a field, and let $A \in \mathbb{F}^{n \times m}$.

(a) Suppose $\text{rank}(A) = 0$.

- What is A ?
- What is a basis for $\text{Col}(A)$?
- What is a basis for $\text{Row}(A)$?
- What is $\text{Nul}(A)$?
- What is one (simplest possible) basis of $\text{Nul}(A)$?

(b) If $\dim(\text{Nul}(A)) = 0$, what is one (simplest possible) basis of $\text{Col}(A)$?

Exercise 4. Determine which (if any) of the following are bases of \mathbb{R}^4 .
(**Remark/Hint:** The numbers are ugly. Can you avoid doing any messy computations?)

$$(a) B_1 = \left\{ \begin{bmatrix} 1 \\ \sqrt{5} \\ -1/\sqrt{13} \\ 3 \end{bmatrix}, \begin{bmatrix} 3/17 \\ 0 \\ 17/3 \\ \pi \\ \pi^2 \end{bmatrix}, \begin{bmatrix} 1 \\ \log_2 3 \\ \log_2 7 \\ \log_2 11 \end{bmatrix} \right\}$$

$$(b) B_2 = \left\{ \begin{bmatrix} 1 \\ \sqrt{5} \\ -1/\sqrt{13} \\ 3 \end{bmatrix}, \begin{bmatrix} 3/17 \\ 0 \\ 17/3 \\ \pi \\ \pi^2 \end{bmatrix}, \begin{bmatrix} 1 \\ \log_2 3 \\ \log_2 7 \\ \log_2 11 \end{bmatrix}, \begin{bmatrix} 1 \\ -\pi \\ 2 \\ \pi \\ 3 \end{bmatrix}, \begin{bmatrix} \pi \\ 1 \\ \pi \\ 2 \\ \pi \end{bmatrix} \right\}$$

$$(c) B_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ \pi^2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} \pi \\ \pi^2 \\ \pi^3 \\ 4 \end{bmatrix} \right\}$$