Linear Algebra 1: Tutorial 5

Irena Penev

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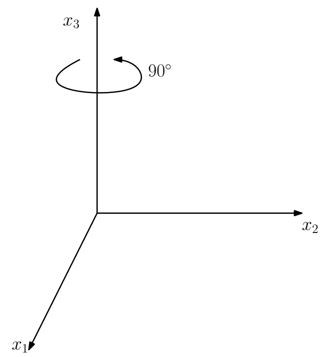
Problem 1 from HW#2. *Solve the linear system below, where k is some fixed constant. (The coefficients are assumed to be in $\mathbb{R}$.)*

$$\begin{array}{ccccccc} x & + & y & - & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ x & + & y & + & (k^2-5)z & = & k \end{array}$$

> ***Remark:*** *Your solutions will depend on k. In fact, the* ***number*** *of solutions will also depend on k. You will need to figure out for which (if any) k the system has no solutions, for which it has a unique solution, and for which it has infinitely many solutions.*

Exercise 3 from Tutorial 4. *Find the standard matrices of the following linear transformations (you may assume they are linear).*

(a) $f : \mathbb{R}^3 \to \mathbb{R}^3$ that rotates each vector about the x_3 axis by 90° counterclockwise.

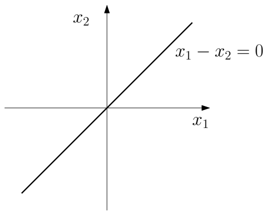


(b) $g : \mathbb{R}^3 \to \mathbb{R}^3$ that projects each vector onto the x_1x_3-plane.

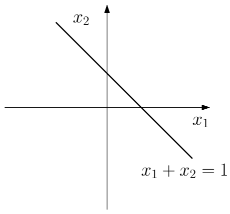
(c) $h_1 : \mathbb{R}^3 \to \mathbb{R}^3$ that first rotates each vector about the x_3 axis by 90° counterclockwise, and then projects each vector onto the x_1x_3-plane.

(d) $h_2 : \mathbb{R}^3 \to \mathbb{R}^3$ *that first projects each vector onto the* x_1x_3*-plane, and then rotates each vector about the* x_3 *axis by 90° counterclockwise.*

Exercise 4 from Tutorial 4. *Find the standard matrix of the linear transformation* $f : \mathbb{R}^2 \to \mathbb{R}^2$ *that projects each vector onto the line given by the equation* $x_1 - x_2 = 0$ *(you may assume the transformation is linear).*



Exercise 5 from Tutorial 4. *Explain why the function* $g : \mathbb{R}^2 \to \mathbb{R}^2$ *that projects each vector onto the line* $x_1 + x_2 = 1$ *is* ***not*** *linear.*



Exercise 1. *Let*

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

with entries understood to be in $\mathbb{Z}_2$*. Solve the equation* $AX = B$*. How many solutions does the equation* $AX = B$ *have?*

Exercise 2. *Let*

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \textit{array} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

with entries understood to be in $\mathbb{R}$*. Solve the equation* $AX = B$*. How many solutions does the equation* $AX = B$ *have?*

Exercise 3. *Let*

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \quad \textit{and} \quad B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \end{bmatrix}$$

with entries understood to be in $\mathbb{Z}_3$*. Solve the equation* $XA = B$*. How many solutions does the equation* $XA = B$ *have?*