Linear Algebra 1: Tutorial 4

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Exercise 1. *Determine which of the following functions are linear (and prove your answer is correct). For those that are linear, find their standard matrix.*

(a) $f_1 : \mathbb{R}^2 \to \mathbb{R}^2$, *given by* $f_1\Big(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\Big) = \begin{bmatrix} \frac{1}{2}u_1 + \frac{1}{3}u_2 \\ 3u_1 \end{bmatrix}$ *for all* $u_1, u_2 \in \mathbb{R}$.

(b) $f_2 : \mathbb{R}^3 \to \mathbb{R}^4$, *given by* $f_2\Big(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\Big) = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ 1 \end{bmatrix}$ *for all* $u_1, u_2, u_3 \in \mathbb{R}$.

(c) $f_3 : \mathbb{Z}_2^2 \to \mathbb{Z}_2^2$, *given by* $f_3\Big(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\Big) = \begin{bmatrix} u_1 \\ u_2^2 \end{bmatrix}$ *for all* $u_1, u_2 \in \mathbb{Z}_2$.

(d) $f_4 : \mathbb{Z}_3^2 \to \mathbb{Z}_3^2$, *given by* $f_4\Big(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\Big) = \begin{bmatrix} u_1 \\ u_2^2 \end{bmatrix}$ *for all* $u_1, u_2 \in \mathbb{Z}_3$.

Exercise 2. *For each of the following matrices, determine whether it is invertible, and if so, find its inverse.*

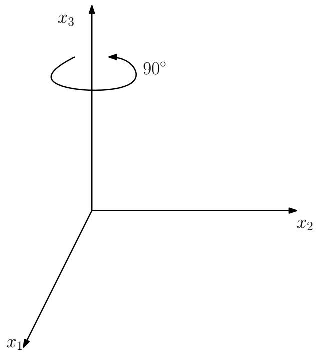
(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$, *with entries understood to be in* $\mathbb{R}$.

(b) $B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_2$.

(c) $C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$, *with entries understood to be in* $\mathbb{Z}_3$.

Exercise 3. *Find the standard matrices of the following linear transformations (you may assume they are linear).*

(a) $f : \mathbb{R}^3 \to \mathbb{R}^3$ *that rotates each vector about the* x_3 *axis by 90° counterclockwise.*

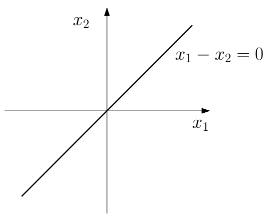


(b) $g : \mathbb{R}^3 \to \mathbb{R}^3$ *that projects each vector onto the* x_1x_3*-plane.*

(c) $h_1 : \mathbb{R}^3 \to \mathbb{R}^3$ *that first rotates each vector about the* x_3 *axis by 90° counterclockwise, and then projects each vector onto the* x_1x_3*-plane.*

(d) $h_2 : \mathbb{R}^3 \to \mathbb{R}^3$ *that first projects each vector onto the* x_1x_3*-plane, and then rotates each vector about the* x_3 *axis by 90° counterclockwise.*

Exercise 4. *Find the standard matrix of the linear transformation* $f : \mathbb{R}^2 \to \mathbb{R}^2$ *that projects each vector onto the line given by the equation* $x_1 - x_2 = 0$ *(you may assume the transformation is linear).*



Exercise 5. *Explain why the function* $g : \mathbb{R}^2 \to \mathbb{R}^2$ *that projects each vector onto the line* $x_1 + x_2 = 1$ *is* ***not*** *linear.*