# Linear Algebra 1:
# HW#5

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due Thursday, November 24, 2022, at 10 am (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Alternatively (if you don't feel like typing or scanning), you may submit a hard copy of your HW in lecture or tutorial **before** the deadline. Please do **not** e-mail your HW to me. Please write your **name** on the top of the first page of your HW.

**Problem 1** (25 points)**.** *Let $\mathbb{F}$ be a field, and let $A \in \mathbb{F}^{n\times n}$ and $\mathbf{b} \in \mathbb{F}^n$. Prove that $A$ is invertible if and only if the matrix-vector equation $A\boldsymbol{x} = \mathbf{b}$ has a unique solution.*

> ***Hint:*** *One direction (which one?) was proven in the Lecture Notes. There is more than one way to prove the other direction, but the easiest way might be to use a result about homogeneous matrix-vector equations proven in the Lecture Notes.*

**Problem 2** (25 points)**.** *Prove or disprove the following statement: "For all matrices $A, B \in \mathbb{R}^{2\times 2}$, if the matrix equation has $XA = B$ has a unique solution, then that solution is an invertible matrix."*

> ***Remark:*** *First, state clearly whether the statement is true or false. If it is true, then prove it. If it is false, then find a counterexample (and prove that your counterexample is correct).*

**Problem 3** (40 points)**.** *Prove parts (b) and (d) of Proposition 2.4 from Lecture Notes 5.*

***Hint:*** *Imitate the proofs of parts (a) and (c).*

**Problem 4** (10 points)**.** *Consider the permutation* $\pi = (165)(2873)$ *in* $S_9$.

*(a) [5 points] Express* $\pi$ *in the table form, i.e. fill in the blanks below.*

$$\pi \quad = \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ & \_ \end{pmatrix}$$

*(b) [5 points] Compute* $sgn(\pi)$.