Linear Algebra 1:
HW#2

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due Thursday, October 27, 2022, at 10 am (Prague time)

Submit your HW through the **Postal Owl** as a **PDF attachment**. Make sure your submission is **printable**: it should be A4 or letter size, and written in dark ink/pencil (blue, black...) on a light (white, beige...) background. Other formats will not be accepted. Alternatively (if you don't feel like typing or scanning), you may submit a hard copy of your HW in lecture or tutorial **before** the deadline. Please do **not** e-mail your HW to me. Please write your **name** on the top of the first page of your HW.

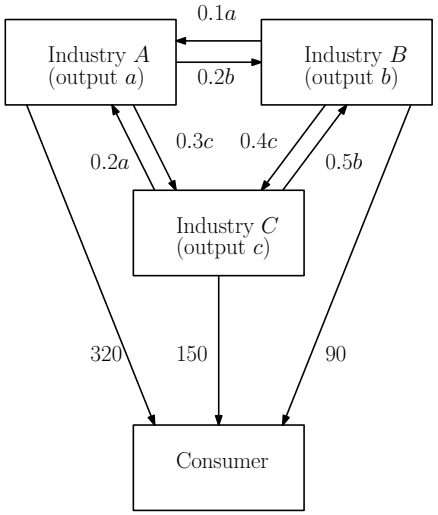
Problem 1 (25 points)**.** *Solve the linear system below, where k is some fixed constant. (The coefficients are assumed to be in $\mathbb{R}$.)*

$$\begin{array}{rcrcrcl} x & + & y & - & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ x & + & y & + & (k^2-5)z & = & k \end{array}$$

> ***Remark:*** *Your solutions will depend on k. In fact, the* ***number*** *of solutions will also depend on k. You will need to figure out for which (if any) k the system has no solutions, for which it has a unique solution, and for which it has infinitely many solutions.*

Problem 2 (25 points)**.** *Let a and b be real constants. For what values of the real constant c is the vector* $\begin{bmatrix} 1 \\ c \\ c^2 \end{bmatrix}$ *a linear combination of the vectors* $\begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$ *and* $\begin{bmatrix} 1 \\ b \\ b^2 \end{bmatrix}$*? Make sure you prove that your answer is correct.*

Problem 3 (25 points)**.** *An economy has three industries, A, B, and C, which have annual output worth a, b, and c, respectively (this is measured in some units, such as hundreds of millions of dollars). The consumer requires a certain amount of output from each industry (as shown by the arrows). However, in order to produce output, each industry requires input from the other two industries, as shown by the arrows. (For instance, if industry A is to produce total output valued a, it needs to receive input valued $0.1a$ from industry B, as well as input valued $0.2a$ from industry C.) Compute the output values a, b, c so that the requirements of both the consumer and the three industries are satisfied.*



Remark: *The calculation gets messy, and so feel free to use software for row reduction.*[1] *Your task is to set up the linear system and the augmented matrix correctly, and then to read off solutions.*

Problem 4 (25 points)**.** *Consider the chemical reaction*

$$a\ \mathrm{NO_2} + b\ \mathrm{H_2O} \longrightarrow c\ \mathrm{HNO_2} + d\ \mathrm{HNO_3},$$

where a, b, c are unknown positive integers. The reaction must be balanced, that is, the number of atoms of each element must be the same before and after the reaction. For example, because the number of oxygen (O) atoms must remain the same, we have $2a + b = 2c + 3d$. While there are many possible values of a, b, c, d, it is customary to use the smallest possible positive integers. By setting up and solving a linear system, balance the reaction above (i.e. find the appropriate values of a, b, c, d).

[1]Some calculators have this function. If you don't have such a calculator, Google "reduced row echelon form calculator" to find a web site that will compute it.