NDMI012: Combinatorics and Graph Theory 2
Tutorial 14

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Summer 2022

Thursday, May 19

Exercise 1. *For each pair of positive integers, p and ℓ, construct a family $\mathscr{A}$ of $(p-1)^\ell$ non-empty sets, each of size ℓ, such that there does **not** exist a sunflower $\mathscr{S} \subseteq \mathscr{A}$ with p petals.*

Exercise 2. *For each pair of positive integers, n and r, find $ex(n, K_{1,r})$.*

Exercise 3. *Prove that for all positive integers r and n, we have that*

$$t_r(n) \quad \leq \quad \tfrac{r-1}{2r} n^2,$$

and that equality holds whenever r divides n.

> ***Hint:*** *Set $n = kr + \ell$, where k and ℓ are non-negative integers with $\ell \leq r-1$. Treat the case $\ell = 0$ first, and then show for the general case that $t_r(n) = \frac{r-1}{2r}(n^2 - \ell^2) + \binom{\ell}{2}$.*

Exercise 4. *Let r be a positive integer. Prove that*

$$\lim_{n\to\infty} \frac{t_r(n)}{\binom{n}{2}} \quad = \quad \tfrac{r-1}{r}.$$

> ***Hint:*** *$r\lfloor \frac{n}{r} \rfloor \leq n \leq r\lceil \frac{n}{r} \rceil$, and consequently, $t_r\Big(r\lfloor \frac{n}{r} \rfloor\Big) \leq t_r(n) \leq t_r\Big(r\lceil \frac{n}{r} \rceil\Big)$. Use Exercise 3.*