NDMI012: Combinatorics and Graph Theory 2
Tutorial 12

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Exercise 5 from Tutorial 11. *Let G be a Hamiltonian bipartite graph, and let $x, y \in V(G)$. Prove that $G \setminus \{x, y\}$ has a perfect matching if and only if x and y are on the opposite sides of the bipartition of G. Apply this to prove that deleting two unit squares from an 8×8 chessboard leaves a board that can be partitioned into 1×2 rectangles if and only if the two missing squares have opposite colors.*

Exercise 1. *A graph is* cubic *if all its vertices are of degree three. Construct a 2-connected cubic bipartite graph that is not Hamiltonian.*

Exercise 2.

(a) Using Burnside's lemma, find the number of non-isomorphic graphs on four vertices.

> ***Hint:*** *This is similar to (but easier than) Example 3.2 from Lecture Notes 11.*

(b) Draw all non-isomorphic graphs on four vertices. (You do not have to prove that they are non-isomorphic.)

Definition. *For an integer $n \geq 3$, the* dihedral group *D_{2n} is the group of symmetries of the regular n-gon. Its elements are the identity function, $n-1$ rotations about the center of the n-gon (by $\frac{i}{n} \cdot 360°$, for $i \in \{1, \dots, n-1\}$), and n reflections. The group operation is the composition of functions.*

Exercise 3. *Let k be a positive integer, and let P_k be the set of all colorings of the edges of the regular pentagon using the color set $\{1, \dots, k\}$. Two colorings in P_k are* equivalent *if one can be transformed into another by a symmetry in D_{10}. Using Burnside's lemma, compute the number of non-equivalent colorings in P_k.*

> ***Hint:*** *This is similar to (but easier than) Example 3.1 from Lecture Notes 11.*

Exercise 4. *Let k be a positive integer, and let H_k be the set of all colorings of the edges of the regular hexagon using the color set $\{1, \ldots, k\}$. Two colorings in H_k are* equivalent *if one can be transformed into another by a symmetry in D_{12}. Using Burnside's lemma, compute the number of non-equivalent colorings in H_k.*

> ***Hint:*** *This is similar to (but easier than) Example 3.1 from Lecture Notes 11.*