NDMI012: Combinatorics and Graph Theory 2
Tutorial 4

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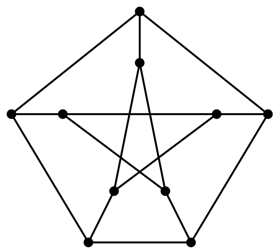
Exercise 2 from Tutorial 3. *Prove that the Petersen graph (below) is non-planar. (You may use the Kuratowski-Wagner theorem.)*



Petersen graph

Exercise 4 from Tutorial 3. *Show that a graph is outerplanar if and only if it contains neither K_4 nor $K_{2,3}$ as a minor.*

> ***Hint:*** *Use the Kuratowski-Wagner theorem.*

Definition. *A graph is* maximally planar *if it is planar, and it is not a proper subgraph of any planar graph on the same vertex set.*[1]

Definition. *A* minimal non-planar graph *is a graph that is not planar, but all of its proper subgraphs are planar.*

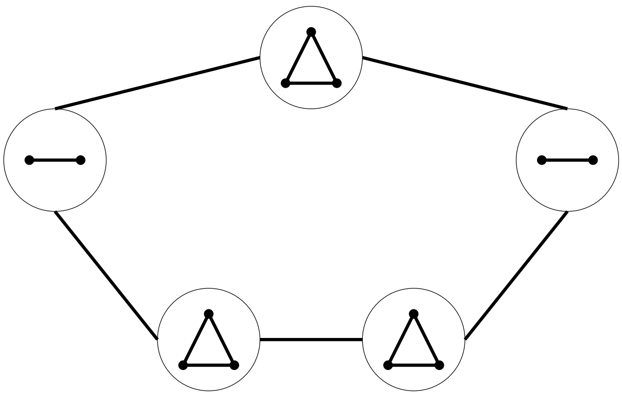
Exercise 5 from Tutorial 3. *Does every minimal non-planar graph G contain an edge e such that $G - e$ is maximally planar? Does the answer change if we define "minimal" with respect to minors rather than subgraphs?*

[1] This means that the graph is planar, but turning any non-edge of the graph into an edge produces a non-planar graph.

Definition. *A graph is called* outerplanar *if it has a drawing in the plane such that all vertices lie on the outer face.*

Exercise 6 from Tutorial 3. *Let G be a 3-connected graph on at least six vertices, and assume that G contains K_5 as a topological minor. Prove that G contains $K_{3,3}$ as a topological minor.*

Exercise 1. *Verify that the graph below is a counterexample to Hajós' Conjecture for $k = 7$,[2] that is, show that the graph has chromatic number 7, but does not contain K_7 as a topological minor. Then, show that this graph is **not** a counterexample to Hadwiger's Conjecture, that is, show that it contains K_7 as a minor.*



[2]A line between two circles indicates that all vertices inside one of the circles are adjacent to all vertices inside the other circle.