NDMI012: Combinatorics and Graph Theory 2
Tutorial 3

Irena Penev
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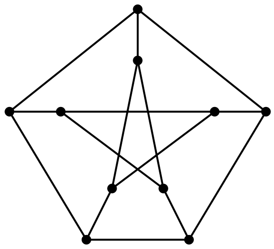
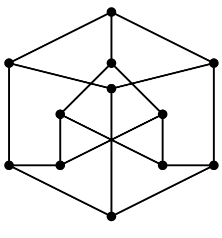
Exercise 1. *Without using the Kuratowski-Wagner theorem, prove that $K_{3,3}$ is not planar.*

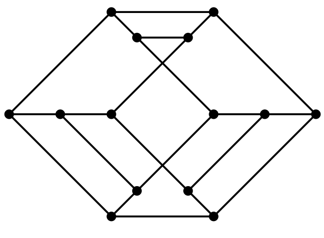
Exercise 2. *Prove that the Petersen graph (below) is non-planar. (You may use the Kuratowski-Wagner theorem.)*



Petersen graph

Exercise 3. *Determine whether the graphs below are planar.*





Exercise 4. *Show that a graph is outerplanar if and only if it contains neither K_4 nor $K_{2,3}$ as a minor.*

Hint: *Use the Kuratowski-Wagner theorem.*

Definition. *A graph is* maximally planar *if it is planar, and it is not a proper subgraph of any planar graph on the same vertex set.*[1]

Definition. *A* minimal non-planar graph *is a graph that is not planar, but all of its proper subgraphs are planar.*

Exercise 5. *Does every minimal non-planar graph G contain an edge e such that $G - e$ is maximally planar? Does the answer change if we define "minimal" with respect to minors rather than subgraphs?*

Definition. *A graph is called* outerplanar *if it has a drawing in the plane such that all vertices lie on the outer face.*

Exercise 6. *Let G be a 3-connected graph on at least six vertices, and assume that G contains K_5 as a topological minor. Prove that G contains $K_{3,3}$ as a topological minor.*

[1]This means that the graph is planar, but turning any non-edge of the graph into an edge produces a non-planar graph.