# NDMI012: Combinatorics and Graph Theory 2
Tutorial 2

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Reminder: A *maximum matching* in a graph G is a matching of size $\nu(G)$. On the other hand, a *maximal matching* in G is a matching that is not a proper subset of any other matching of G.

Exercise 6 from Tutorial 1. *Prove that a tree T has a perfect matching if and only if $odd(T \setminus v) = 1$ for every vertex $v \in V(T)$.*

Exercise 7 from Tutorial 1. *Find all perfect matchings of the Petersen graph (shown below). Make sure you prove that the perfect matchings that you found are the only ones.*
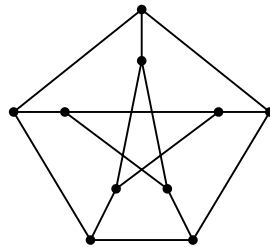
Petersen graph

Exercise 1. *Prove that every maximal matching in a graph G has at least $\frac{\nu(G)}{2}$ many edges.*

Exercise 2. *Let M_0 be a matching in a graph G, and let u be a vertex of G that is unsaturated by M_0. Assume that no M_0-augmenting path of G starts at u. Prove that u is unsaturated by some maximum matching of G.*

Exercise 3. *Prove that any cubic graph with at most two bridges has a perfect matching.*