

NDMI012: Combinatorics and Graph Theory 2

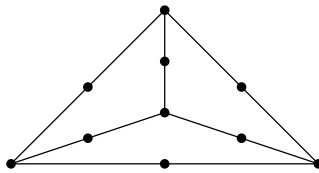
Tutorial 1

Irena Penev
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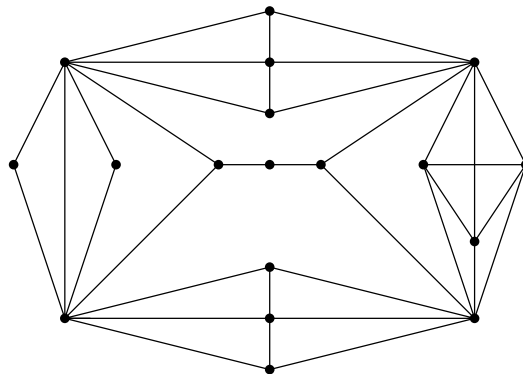
Thursday, February 17

Exercise 1. A maximal matching in a graph G is a matching M such that M is not a proper subset of any other matching of G .¹ Give an example of a graph G and a matching M such that M is a maximal, but not a maximum matching of G .

Exercise 2. Either find a perfect matching in the graph below, or show that the graph does not have a perfect matching.



Exercise 3. Find a maximum matching in the graph below, and give a short proof that your matching is indeed maximum.



¹This means that there is no matching M' of G such that $M \subsetneq M'$.

Exercise 4. Using Tutte's theorem, prove that a graph G is hypomatchable if and only if $|V(G)|$ is odd and all non-empty sets $S \subseteq V(G)$ satisfy $|S| \geq \text{odd}(G \setminus S)$.

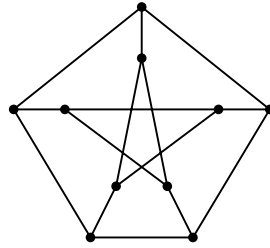
Hint: How is the parity of $|S|$ related to the parity of $\text{odd}(G \setminus S)$?

Exercise 5. Prove or disprove the following statement: "Every forest has at most one perfect matching."

Remark: So, if the statement is true, then you should prove it. If it is false, then you should construct a counterexample.

Exercise 6. Prove that a tree T has a perfect matching if and only if $\text{odd}(T \setminus v) = 1$ for every vertex $v \in V(T)$.

Exercise 7. Find all perfect matchings of the Petersen graph (shown below). Make sure you prove that the perfect matchings that you found are the only ones.



Petersen graph