NDMI012: Combinatorics and Graph Theory 2
HW#10

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due Thursday, May 12, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Definition. *For an integer $n \geq 3$, the* dihedral group D_{2n} *is the group of symmetries of the regular n-gon. Its elements are the identity function, $n-1$ rotations about the center of the n-gon (by $\frac{i}{n} \cdot 360°$, for $i \in \{1, \dots, n-1\}$), and n reflections. The group operation is the composition of functions.*

Problem 1 (50 points)**.** *Let k be a positive integer, and let H_k be the set of all colorings of the edges of the regular hexagon using the color set $\{1, \dots, k\}$. Two colorings in H_k are* equivalent *if one can be transformed into another by a symmetry in D_{12}. Using Burnside's lemma, compute the number of non-equivalent colorings in H_k.*

> ***Hint:*** *This is similar to (but easier than) Example 3.1 from Lecture Notes 11.*

Problem 2 (50 points)**.** *Let R_{cube} be the group of rotations of the cube, as in Example 2.3 of Lecture Notes 11, and let k be a positive integer. Let V_k be the set of all colorings of the* ***vertices*** *of the cube using the color set $\{1, \dots, k\}$.*[1] *Then R_{cube} acts on the set V_k in the natural way: a rotation in R_{cube} maps each element of V_k to an appropriately rotated coloring. Two colorings in V_k are* equivalent *if one can be transformed into the other by a rotation in R_{cube}. Using Burnside's lemma, compute the number of non-equivalent colorings in V_k.*

[1] Remark: In Example 3.1 of Lecture Notes 11, we considered **face** colorings. Here, we consider **vertex** colorings. (These colorings are simply assignments of colors to the vertices. They need not be proper colorings of the cube, viewed as a graph.)