NDMI012: Combinatorics and Graph Theory 2
HW#7

Irena Penev
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due Thursday, April 14, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Problem 1 (25 points)**.** *Without using the Perfect Graph Theorem (or any other result of Lecture Notes 8), prove that complements of chordal graphs are perfect.*

> ***Hint:*** *This is almost immediate from one of the results of Lecture Notes 7. Which one?*

Problem 2 (25 points)**.** *Let G be a chordal graph, and let $e = xy$ be an edge that belongs to some cycle of G. Prove that there exists a vertex $z \in V(G) \setminus \{x, y\}$ such that $\{x, y, z\}$ is a triangle in G.*

Problem 3 (50 points)**.** *Let G be a chordal graph, and let Q be a maximal clique in G.*[1] *Prove that at least one of the following holds:*

(a) Q is a clique-cutset of G;

(b) some vertex $v \in Q$ is simplicial in G.

[1] maximal = inclusion-wise maximal