

NDMI012: Combinatorics and Graph Theory 2

HW#5

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due Thursday, March 31, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Problem 1 (50 points). *Let G be a graph whose odd cycles are pairwise intersecting, i.e. every two odd cycles of G share at least one vertex. Prove that $\chi(G) \leq 5$.*

Problem 2 (50 points). *For all positive integers k , construct a tree T_k of maximum degree k and an ordering of $V(T_k)$ such that the greedy algorithm applied to T_k with that ordering uses $k + 1$ colors. Make sure you prove that your construction is correct.*

Hint: *Use induction and construct the tree and ordering simultaneously.*