

NDMI012: Combinatorics and Graph Theory 2

HW#4

Irena Penev
Summer 2022

due Thursday, March 24, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Definition. A graph is maximally planar if it is planar, and it is not a proper subgraph of any planar graph on the same vertex set.¹

Definition. A minimal non-planar graph is a graph that is not planar, but all of whose proper subgraphs are planar.

Problem 1 (40 points). Does every minimal non-planar graph G contain an edge e such that $G - e$ is maximally planar? Prove that your answer is correct.

Definition. For graphs G_1 and G_2 , the join of G_1 and G_2 , denoted by $G_1 \vee G_2$, is the graph obtained from the disjoint union of G_1 and G_2 by adding all possible edges between G_1 and G_2 .

Problem 2 (30 points). Prove that $C_4 \vee C_4$ cannot be drawn on the projective plane (without edge crossings).

Problem 3 (30 points). Draw $K_{4,4}$ on the Klein bottle (without edge crossings). For this, you should use the polygonal representation of the Klein bottle (i.e. $AABB$ or $ABAB^{-1}$, whichever you find more convenient).

¹This means that the graph is planar, but turning any non-edge of the graph into an edge produces a non-planar graph.