NDMI012: Combinatorics and Graph Theory 2
HW#2

Irena Penev
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due Thursday, March 3, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Definition. *A* vertex cover *of a graph G is a set $C \subseteq V(G)$ such that every edge of G has at least one endpoint in C. The* vertex cover number *of G, denoted by $\tau(G)$, is the smallest size of a vertex cover of G, i.e. $\tau(G) := \min\{|C| \mid C \text{ is a vertex cover of } G\}$.*

Problem 1 (50 points)**.**

(a) [25 points] Prove that every graph G satisfies $\tau(G) \leq 2\nu(G)$.

(b) [25 points] For each positive integer k, construct a graph G_k such that $\nu(G) = k$ and $\tau(G) = 2k$. Make sure you prove that your construction is correct.

Problem 2 (50 points)**.** *Let M_0 be a matching in a graph G, and let u be a vertex of G that is unsaturated by M_0. Assume that no M_0-augmenting path of G starts at u. Prove that u is unsaturated by some maximum matching of G.*