

NDMI012: Combinatorics and Graph Theory 2

HW#1

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due Thursday, February 24, 2022, 15:40 (at the beginning of the tutorial)

Remark: Bring your HW to the beginning of the tutorial. If you must miss the tutorial, please e-mail your HW to me (ipenev@iuuk.mff.cuni.cz) as a **PDF attachment** (no other format will be accepted).

Problem 1 (50 points). *Let $d \geq 3$ be an odd integer, and let G be a d -regular and $(d - 1)$ -edge-connected graph. Prove that G has a perfect matching.*

***Hint:** Imitate the proof Petersen's theorem.*

Problem 2 (50 points). *Using Tutte's theorem, prove the Tutte-Berge formula. You may use Remark 3.1 from Lecture Notes 1, but you may **not** use Lemmas 3.2 and 3.3.*

***Remark:** In lecture, we proved Tutte's theorem using the Tutte-Berge formula. Here, you are supposed to assume Tutte's theorem, and then prove the Tutte-Berge formula.*

***Hint:** Add a suitable number of universal vertices to your graph,¹ and use Tutte's theorem to show that the graph that you obtain has a perfect matching. Then what?*

¹A *universal vertex* is a vertex that is adjacent to all other vertices of the graph.