

NDMI012: Combinatorics and Graph Theory 2

HW#10

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due Tuesday, June 1, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Remark: You are asked to solve the following problems using **Burnside's lemma**. There are, in principle, other ways to solve these problems. However, you are specifically asked to use Burnside's lemma.

Problem 1 (30 points). *Using the **Pólya enumeration theorem**, find the number of non-equivalent colorings of a 4-bead necklace using colors red, blue, and green, if **all three** colors must be used. Two colorings are equivalent if one can be transformed into the other via a **rotation** (reflections do not count).*

Problem 2 (30 points). *Using the **Pólya enumeration theorem**, for each non-negative integer k , find the number of non-isomorphic four-vertex graphs with k edges.*

Problem 3 (40 points). *Solve the following problems using **generating functions** (ordinary or exponential, as appropriate).*

- (a) [20 points] *Find the number of ways that three letters from the word BANANA can be selected (order does not matter).*
- (b) [20 points] *Find the number of ways that three letters from the word BANANA can be arranged (order matters).*

In both parts, the two N's are considered the same (for example, if we select exactly one N, it does not matter which particular one got selected). The same holds for the three A's.