NDMI012: Combinatorics and Graph Theory 2
HW#9

Irena Penev
Winter 2020/2021

due Tuesday, May 25, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Remark: You are asked to solve the following problems using **Burnside's lemma**. There are, in principle, other ways to solve these problems. However, you are specifically asked to use Burnside's lemma.

Problem 1 (40 points)**.**

(a) [30 points] Using Burnside's lemma, find the number of non-isomorphic graphs on four vertices.

> ***Hint:*** *This is similar to (but easier than) Example 3.2 from Lecture Notes 11.*

(b) [10 points] Draw all non-isomorphic graphs on four vertices. (You do not have to prove that they are non-isomorphic.)

Definition. *For an integer $n \geq 3$, the* dihedral group D_{2n} *is the group of symmetries of the regular n-gon. Its elements are the identity function, $n-1$ rotations about the center of the n-gon (by $\frac{i}{n} \cdot 360^\circ$, for $i \in \{1, \dots, n-1\}$), and n reflections. The group operation is the composition of functions.*

Problem 2 (30 points)**.** *Let k be a positive integer, and let P_k be the set of all colorings of the edges of the regular pentagon using the color set $\{1, \dots, k\}$. Two colorings in P_k are* equivalent *if one can be transformed into another by a symmetry in D_{10}. Using Burnside's lemma, compute the number of non-equivalent colorings in P_k.*

Hint: *This is similar to (but easier than) Example 3.1 from Lecture Notes 11.*

Problem 3 (30 points)**.** *Let k be a positive integer, and let H_k be the set of all colorings of the edges of the regular hexagon using the color set $\{1, \ldots, k\}$. Two colorings in H_k are* equivalent *if one can be transformed into another by a symmetry in D_{12}. Using Burnside's lemma, compute the number of non-equivalent colorings in H_k.*

Hint: *This is similar to (but easier than) Example 3.1 from Lecture Notes 11.*