

NDMI012: Combinatorics and Graph Theory 2

HW#4

Irena Penev
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due Tuesday, April 6, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (50 points). *Prove that for every graph G , there exists an ordering $v_1, \dots, v_n$ of the vertices of G such that the greedy coloring algorithm applied to G with the ordering $v_1, \dots, v_n$ yields an optimal coloring of G .¹*

Problem 2 (50 points). *Using the theorem below, prove that every bipartite graph G satisfies $\chi'(G) = \Delta(G)$.²*

Theorem. *Every regular bipartite graph that has at least one edge has a perfect matching.³*

¹An *optimal coloring* of G is a proper coloring of G that uses precisely $\chi(G)$ colors.

²We already proved this in Lecture Notes 5. Here, you are asked to give a different proof.

³This theorem was proven in Combinatorics & Graph Theory 1.