NDMI012: Combinatorics and Graph Theory 2
HW#3

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due Tuesday, March 30, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Definition 1. *Given a graph G and disjoint sets $A, B \subseteq V(G)$,*

- *we say that A is* complete *to B if every vertex of A is adjacent to every vertex of B;*[1]
- *we say that A is* anticomplete *to B if no vertex of A is adjacent to any vertex of B.*[2]

Problem 1 (60 points)**.** *Let G be a graph whose vertex set can be partitioned into five sets, call them A_0, A_1, A_2, A_3, A_4 (with indices understood to be in $\mathbb{Z}_5$), satisfying the following properties:*

- $|A_0| = |A_2| = |A_3| = 3$ *and* $|A_1| = |A_4| = 2$;
- A_0, A_1, A_2, A_3, A_4 *are all cliques;*
- *for all $i \in \mathbb{Z}_5$, A_i is complete to $A_{i-1} \cup A_{i+1}$ and anticomplete to $A_{i-2} \cup A_{i+2}$.*

(So, G is precisely Catlin's graph from Lecture Notes 4.)

(a) [20 points] Prove that $\chi(G) = 7$;

(b) [20 points] Prove that $K_7 \not\preceq_t G$;

(c) [20 points] Prove that $K_7 \preceq_m G$.

[1]So, all possible edges between A and B are present.
[2]So, there are no edges between A and B.

Definition 2. *A graph G is* maximally planar *if it is planar, but for all distinct, non-adjacent vertices $x, y \in V(G)$, the graph $G + xy$ is not planar.*[3]

Definition 3. *A* minimally non-planar *graph is a graph that is not planar, but all of whose proper subgraphs are planar.*

Problem 2 (40 points)**.** *Prove or disprove the following statement: "Every minimally non-planar graph G has an edge e such that $G - e$ is maximally planar."*

> ***Remark:*** *So, if the statement is correct, then you should prove it. If it is false, you should construct a counterexample (and prove that your counterexample is correct).*

[3]Here, $G + xy$ is the graph with vertex set $V(G)$ and edge set $E(G) \cup \{xy\}$. So, $G + xy$ is the graph obtained from G by adding an edge between x and y.