

NDMI012: Combinatorics and Graph Theory 2

HW#2

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Winter 2020/2021

due Tuesday, March 25, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (40 points). Let M_0 be a matching in a graph G , and let u be a vertex of G that is unsaturated by M_0 . Assume that no M_0 -augmenting path of G starts at u . Prove that u is unsaturated by some maximum matching of G .

Problem 2 (20 points). Prove that $K_{3,3}$ is not planar.

Hint: Imitate the proof for K_5 .

Problem 3 (40 points). A graph is outerplanar if it can be drawn in the plane so that all its vertices belong to the boundary of the outer face of the drawing.¹ Using Kuratowski's theorem, prove that a graph is outerplanar if and only if it contains neither K_4 nor $K_{2,3}$ as a minor.

Hint: What can you add (vertices/edges) to a graph so that the old graph is outerplanar if and only if the new one is planar?

¹Equivalently, a graph is *outerplanar* if it can be drawn on the sphere so that all its vertices belong to the boundary of some face of the drawing. For example, the graph below is outerplanar.

