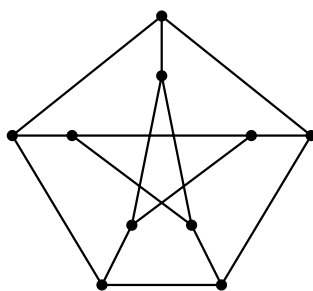# NDMI011: Combinatorics and Graph Theory 1

Tutorial #5

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November 25, 2021

Exercise 1. *Find all perfect matchings in the Petersen graph (represented below). Prove that the graph has no other perfect matchings.*

Petersen graph

Exercise 2. *Using Menger's theorem (any version you like), prove the Kőnig-Egerváry theorem.*

Exercise 3. *Let G be a connected graph with at least one edge, and in which all vertices have even degree.*

  (a) *Must G be 2-connected? Either prove that it must be, or find a counterexample.*

  (b) *Must G be 2-edge-connected? Either prove that it must be, or find a counterexample.*

Exercise 4. *Let $k \geq 2$ be an integer. Prove that if a connected bipartite graph is k-regular, then it is 2-connected. What about k-regular non-bipartite graphs?*