NDMI011: Combinatorics and Graph Theory 1
HW#8

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Winter 2021/2022

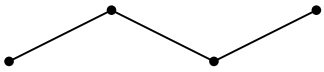
due Wednesday, December 15, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (30 points)**.** *Using the definition of Ramsey numbers from Lecture Notes 10, prove that for all positive integers k and ℓ, we have that $R(k, \ell) = R(\ell, k)$.*

Problem 2 (30 points)**.** *Using Ramsey numbers, prove that for all graphs H_1 and H_2, there exists a positive integer n such that for every graph G on at least n vertices, either G contains H_1 as a subgraph or $\overline{G}$ (the complement of G) contains H_2 as a subgraph.*

Problem 3 (40 points)**.** *As usual, P_4 is the path on four vertices and three edges.*



P_4

(a) *[30 points] Prove that for every graph G on at least five vertices, at least one of $G, \overline{G}$ contains P_4 as a subgraph.*

(a) *[10 points] Show that the bound from part (a) cannot be improved. More precisely, construct a 4-vertex graph G such that neither G nor $\overline{G}$ contains P_4 as a subgraph.*