# NDMI011: Combinatorics and Graph Theory 1
# HW#7

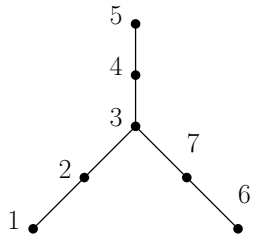
Irena Penev
Winter 2021/2022

due Wednesday, December 8, 2021 before midnight (Prague time)

**Remark:** Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

**Problem 1** (20 points)**.**

*(a) [10 points] Find the Prüfer code of the tree below.*




*(b) [10 points] Find the tree on the vertex set* $\{1, 2, 3, 4, 5, 6, 7\}$ *with Prüfer code* $4, 4, 2, 2, 4$*.*

**Problem 2** (40 points)**.** *Using determinants, compute the number of spanning trees of complete bipartite graph* $K_{m,n}$*.*

**Problem 3** (40 points)**.** *Let* $n, t \in \mathbb{N}$*, with* $t \geq 2$*. Prove that any* $n$*-vertex graph that does not contain the complete bipartite graph* $K_{2,t}$ *as a subgraph has at most* $\frac{1}{2}(n + n^{3/2}\sqrt{t-1})$ *edges.*

> ***Hint:*** *Imitate the proof of Theorem 2.1 from Lecture Notes 9. (Note that* $C_4 = K_{2,2}$*.) If you define* $M$ *the same way as in that proof, you should get a different upper bound for* $|M|$*. Now work with that upper bound.*