NDMI011: Combinatorics and Graph Theory 1
HW#6

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due Wednesday, December 1, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (40 points)**.** *Let $k \geq 2$ be an integer. Prove that if a connected bipartite graph is k-regular, then it is 2-connected.*

A *bridge* in a graph G is an edge $e \in E(G)$ such that $G \setminus e$ has more components than G. Note that a graph is 2-edge-connected if and only if it has at least two vertices, is connected, and does not contain a bridge.[1]

Problem 2 (20 points)**.** *Let G be a connected graph on at least two vertices. Prove that G is 2-edge-connected if and only if every edge of G belongs to a cycle of G.*

Recall that a *path addition* to a graph H is the addition to H of a path between two distinct vertices of H in such a way that no internal vertex and no edge of the path belongs to H. Similarly, a *cycle addition* to a graph H is the addition to H of a cycle such that exactly one vertex and no edge of the cycle belongs to H.

Problem 3 (40 points)**.** *Prove that a graph G is 2-edge-connected if and only if it either is a cycle or can be obtained from a cycle by a sequence of path and cycle additions.*

> ***Hint:*** *Imitate the proof of the Ear Lemma. You may use the statement of Problem 2 even if you did not prove it.*

[1] Depending on how you write up your solutions of the next two problems, you may or may not want to refer to this characterization of 2-connected graphs. (Note that the characterization is immediate from the definition.)