NDMI011: Combinatorics and Graph Theory 1
HW#5

Irena Penev
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due Wednesday, November 24, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (40 points)**.** *Let (G, s, t, c) be a network that has more than one maximum flow. Prove that (G, s, t, c) has infinitely many maximum flows.*

Problem 2 (30 points)**.** *Let G be a bipartite graph with bipartition (A, B), and assume that all non-empty sets $A' \subseteq A$ satisfy $|A'| < |N_G(A')|$. Prove that every edge of G belongs to some A-saturating matching of G.*

Problem 3 (30 points)**.** *Let G be a bipartite graph that has at least one edge. Prove G has a matching of size at least $\lceil |E(G)|/\Delta(G) \rceil$.*

Hint: *The Kőnig-Egerváry theorem.*