NDMI011: Combinatorics and Graph Theory 1
HW#4

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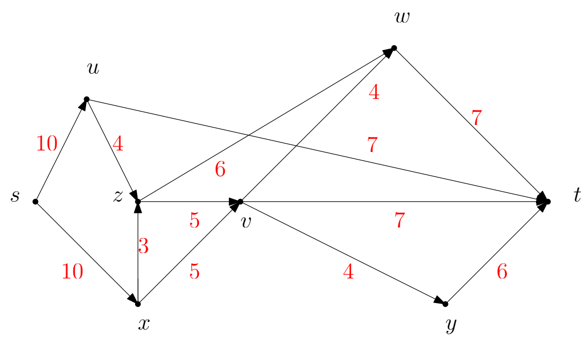
Winter 2021/2022

due Wednesday, November 10, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (50 points). *Using the Ford-Fulkerson algorithm, find a maximum flow and a minimum capacity cut in the network below. (As usual, s is the sourse, t is the sink, and the capacities are in red.) The level of detail in your solution should be comparable to that in Example 3.3 from Lecture Notes 6.*



Problem 2 (50 points). *Let $\mathbb{F}$ be a finite field with n elements, call them $t_0, t_1, \dots, t_{n-1}$, where $t_0 = 0$ and $t_1 = 1$.*[1] *For each $k = 1, \dots, n-1$, we define the $n \times n$ matrix L_k as follows: the (i,j)-th entry*[2] *of L_k is $t_i t_k + t_j$. Prove that $L_1, \dots, L_{n-1}$ are pairwise orthogonal Latin squares.*

> ***Remark:*** *Via Theorem 2.3 from Lecture Notes 5, this yields another construction of a finite projective plane of order n.*

[1] That is: t_0 is the additive identity and t_1 the multiplicative identity of the field $\mathbb{F}$. Note that n must be a power of a prime, since $\mathbb{F}$ is a finite field (but you will not need this fact).

[2] That is: the entry in the i-th row and j-th column. Note that the elements of the field $\mathbb{F}$ are called $t_0, t_1, \dots, t_{n-1}$; accordingly, the rows and columns of the matrices $L_1, \dots, L_{n-1}$ are indexed by $0, 1, \dots, n-1$ (and not by $1, 2, \dots, n$).