NDMI011: Combinatorics and Graph Theory 1
HW#3

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due Wednesday, October 20, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (60 points). *Let (P0), (P1), and (P2) be as in the definition of a finite projective plane. Let $(X, \mathcal{P})$ be a set system such that X is a non-empty, finite set. Prove that statements (1) and (2) below are equivalent.*

*(1) $(X, \mathcal{P})$ satisfies (P1) and (P2), but does **not** satisfy (P0);*

(2) one of the following holds:

- $\mathcal{P} = \{X\}$,
- $|X| = 1$ *and* $\mathcal{P} = \emptyset$,
- $|X| = 1$ *and* $\mathcal{P} = \{\emptyset\}$,
- $|X| \geq 2$ *and there exists some* $a \in X$ *such that* $\mathcal{P} = \{X, \{a\}\}$,
- $|X| \geq 3$ *and there exists some* $a \in X$ *such that* $\mathcal{P} = \{X \setminus \{a\}\} \cup \{\{x, a\} \mid x \in X \setminus \{a\}\}$.

Hint: *One of the exercises from Tutorial 2 (which one?) may be helpful. You may use the statements of tutorial exercises without proving them.*

Problem 2 (40 points). *Prove Theorem 1.4(c) from Lecture Notes 4 directly, i.e. **without** using duality. The theorem is stated below, for your reference. (You may use parts (a) and/or (b) of Theorem 1.4 if you find them useful.)*

Theorem 1.4 from Lecture Notes 4. *Let $(X, \mathcal{P})$ be a finite projective plane of order n. Then all the following hold:*

(a) for each point $x \in X$*, exactly* $n+1$ *lines in* $\mathcal{P}$ *pass through* x*;*

(b) $|X| = n^2 + n + 1$*;*

(c) $|\mathcal{P}| = n^2 + n + 1$*.*