NDMI011: Combinatorics and Graph Theory 1
HW#2

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due Wednesday, October 13, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (30 points)**.** *Find the generating functions of the sequences below. Your final answer should* ***not*** *involve any infinite sums.*

(a) $1, 0, 1, 1, 0, 2, 1, 0, 4, 1, 0, 8, 1, 0, 16, 1, 0, 32, \ldots$

(b) $1, -1, 0, 1, -1, 0, 1, -1, 0, \ldots$

Problem 2 (40 points)**.** *Let S be the set of all strings using the alphabet $\{a, b, c, d\}$, and in which letters a and b never appear next to each other. Using generating functions, find a closed formula for the number of n-letter strings in S.*

> ***Hint:*** *Let u_n be the number of n-letter strings in S that start with a or b, and let v_n be the number of all n-letter strings in S that start with c or d. Find recursive formulas relating the u_n's and v_n's, compute the corresponding generating functions, and then find the formula that you need. Your final answer should somehow involve $\sqrt{17}$.*

Problem 3 (30 points)**.** *There are $2n$ people wishing to buy a \$50 ticket to a Broadway show. Half of these people wish to pay with a \$50 bill, and the other half wish to pay with a \$100 bill (and get a \$50 change). At the beginning, the cashier has no cash. How many ways are there to order these $2n$ people in a line, so that each of them can buy a ticket (and receive change, if appropriate)?*

> ***Hint:*** *This is somehow related to binary trees or perhaps walks on chessboards (see Tutorial 1).*