NDMI011: Combinatorics and Graph Theory 1
HW#10

Irena Penev
Winter 2020/2021

due Monday, January 4, 2021 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (50 points)**.** *Using Ramsey numbers, show that for all* $k \in \mathbb{N}$*, there exists some* $N \in \mathbb{N}$*, such that for all* $n \in \mathbb{N}$ *with* $n \geq N$*, every* n*-term sequence of real numbers contains a* k*-term subsequence that is either strictly increasing or strictly decreasing or constant.*

Problem 2 (50 points)**.** *Let* $n, t \in \mathbb{N}$*, with* $t \geq 2$*. Prove that any* n*-vertex graph that does not contain the complete bipartite graph* $K_{2,t}$ *as a subgraph has at most* $\frac{1}{2}(n + n^{3/2}\sqrt{t-1})$ *edges.*

> ***Hint:*** *Imitate the proof of Theorem 2.1 from Lecture Notes 12. (Note that* $C_4 = K_{2,2}$*.) If you define* M *the same way as in that proof, you should get a different upper bound for* $|M|$*. Now work with that upper bound.*