NDMI011: Combinatorics and Graph Theory 1
HW#9

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Winter 2020/2021

due Monday, December 14, 2020 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (30 points). *Let k and ℓ be positive integers. Using the definition of Ramsey numbers, prove that $R(k, \ell) = R(\ell, k)$.*

Problem 2 (70 points). *Let n be a non-negative integer, and let X be an n-element set. A* half-antichain *in $(\mathscr{P}(X), \subseteq)$ is a set $\mathcal{A}$ of subsets of X such that there do* ***not*** *exist sets $A_1, A_2, A_3 \in \mathcal{A}$ such that $A_1 \subsetneqq A_2 \subsetneqq A_3$.*

(a) [40 points] Prove that any half-antichain in $(\mathscr{P}(X), \subseteq)$ has at most $2\binom{n}{\lfloor n/2 \rfloor}$ elements.

> ***Hint:*** *Imitate the proof of Sperner's theorem. You may use the statements of Claims 1 and 2 from the proof of Sperner's theorem without (re)proving them.*

(b) [30 points] Prove that if n is odd, then there is a half-antichain in $(\mathscr{P}(X), \subseteq)$ that has precisely $2\binom{n}{\lfloor n/2 \rfloor}$ elements.