NDMI011: Combinatorics and Graph Theory 1
HW#8

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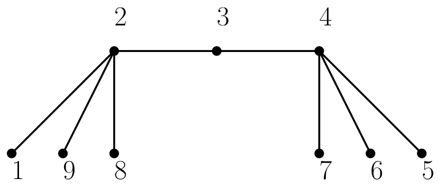
due Monday, December 7, 2020 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (20 points)**.**

(a) [10 points] Find the Prüfer code of the tree below.



(b) [10 points] Find the tree on the vertex set $\{1, \dots, 7\}$*, whose Prüfer code is* $3, 3, 7, 4, 3$.

A *bridge* in a graph G is an edge $e \in E(G)$ such that $G \setminus e$ has more components than G. Note that a graph is 2-edge-connected if and only if it has at least two vertices, is connected, and does not contain a bridge.[1]

Problem 2. *[30 points] Let G be a connected graph on at least two vertices. Prove that G is 2-edge-connected if and only if every edge of G belongs to a cycle of G.*

[1]Depending on how you write up your solutions of the next two problems, you may or may not want to refer to this characterization of 2-connected graphs. (Note that the characterization is immediate from the definition.)

Recall that a *path addition* to a graph H is the addition to H of a path between two distinct vertices of H in such a way that no internal vertex and no edge of the path belongs to H. Similarly, a *cycle addition* to a graph H is the addition to H of a cycle such that exactly one vertex and no edge of the cycle belongs to H.

Problem 3 (50 points)**.** *Prove that a graph G is 2-edge-connected if and only if it either is a cycle or can be obtained from a cycle by a sequence of path and cycle additions.*

> ***Hint:*** *Imitate the proof of the Ear Lemma from Lecture Notes 9. You may use the statement of Problem 2 even if you did not prove it.*