

# NDMI011: Combinatorics and Graph Theory 1

## HW#7

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Winter 2020/2021

due Monday, November 30, 2020 before midnight (Prague time)

**Remark:** Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

**Problem 1** (60 points). *Prove the global version of Menger's theorem.*<sup>1</sup>

**Problem 2** (40 points). *Using Menger's theorem (any version you like), prove the “(a)  $\implies$  (b)” part of the graph theoretic formulation of Hall's theorem. More precisely, let  $G$  be a bipartite graph with bipartition  $(A, B)$ , and assume that all sets  $A' \subseteq A$  satisfy  $|A'| \leq |N_G(A')|$ . Using Menger's theorem, prove that  $G$  has an  $A$ -saturating matching.*

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<sup>1</sup>The statement of this theorem is given at the end of Lecture Notes 8. Part (a) is worth 40 points, and part (b) is worth 20 points.