NDMI011: Combinatorics and Graph Theory 1
HW#1

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due Monday, October 12, 2020 before midnight (Prague time)

Remark: Please e-mail me (ipenev@iuuk.mff.cuni.cz) your HW as a **PDF attachment** (no other format will be accepted).

Problem 1 (100 points)**.** *Use generating functions*[1] *to find a closed formula of the general term of the following recursively defined sequences.*

(a) The sequence $\{a_n\}_{n=0}^{\infty}$ *defined recursively as follows:*

- $a_0 = 2$, $a_1 = 3$;
- $a_{n+2} = 3a_n - 2a_{n+1}$ *for all integers* $n \geq 0$.

(b) The sequence $\{b_n\}_{n=0}^{\infty}$ *defined recursively as follows:*

- $b_0 = 1$;
- $b_{n+1} = 3b_n + 2^{n+1}$ *for all integers* $n \geq 0$.

(c) The sequence $\{c_n\}_{n=0}^{\infty}$ *defined recursively as follows:*

- $c_0 = 1$, $c_1 = 4$;
- $c_{n+2} = -4c_n + 4c_{n+1}$ *for all integers* $n \geq 0$.

(d) The sequence $\{d_n\}_{n=0}^{\infty}$ *defined recursively as follows:*

- $d_0 = 1$;
- $d_{n+1} = d_n + n + 1$ *for all integers* $n \geq 0$.

  Hint for (d): *What do you get when you expand* $\frac{1}{(1+x)^2}$ *and* $\frac{1}{(1-x)^2}$ *as power series? You'll probably need to recognize one of these at some point in your calculation.*

[1] Yes, this can be done in other ways as well. However, you are specifically asked to use generating functions.