Mathematics++

Problem set 4 – Measure Concentration

hints after **May 21, 2018**, solutions due **May 28, 2018**

1. Let B and B' be unit balls in $\mathbb{R}^n$ and let $d(n)$ be the distance between the centers of B and B' so that the probability that a uniformly randomly chosen point of B belongs to B' with probability 1 %. Find some explicit upper bound on $d(n)$ tending to 0 while n tends to infinity. **[3]**

2. Let P be a convex polytope in $\mathbb{R}^n$ which is the intersection of N halfspaces. Let us assume that P contains the unit ball. Show that

$$\lambda(P) \geq \left(C \frac{n}{\ln N} \right)^{n/2} \lambda(B^n)$$

for some constant $C > 0$. *Hint:* Consider a sphere such that the complement of P covers exactly one half of it's surface area. **[5*]**

3. Construct a continuous function $f : S^{n-1} \to [0,1]$, which is not concentrated. That is, $\forall x \in [0,1]$, the set $f^{-1}([x - \frac{1}{3}, x + \frac{1}{3}])$ has measure at most $\frac{2}{3}$. **[3]**